Direct Feedback Linearization of Nonlinear Loudspeaker Systems

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Abstract:
The control law used in regular static state feedback is expressed in a novel direct form which is advantageous for the linearization of loudspeaker systems. Compared with the straightforward integrator-decoupled form the direct form increases the computational efficiency, requires less loudspeaker parameter and can be easily implemented on signal processors working with a fixed-point format. The control law in the direct form can be applied to loudspeaker systems with different acoustic radiation aids (closed-box system, vented-box system) and dispenses with measurement of the internal states and full identification of the acoustic parameters. The relationship between feedback linearization and feedforward filter techniques (volterra filter, mirror filter) is discussed.

1. INTRODUCTION

The nonlinear differential equation of an electrodynamic loudspeaker based on Kaizer's model [1] can also be converted into the general state space form where the control appears linearly (affine) in the dynamics. Hence, regular static state feedback, which is a fundamental technique in the control of affine nonlinear systems, can be used to cope with the dominant nonlinearities in woofer systems and to linearize the overall system. Following this approach, control schemes for woofers have been presented in [2], [3], [4] which only differ in the definition of the states and in the compensation of the nonlinear reluctance force. Each of these feedback controllers comprises two subsystems which are connected in series as shown in Fig. 1. A nonlinear subsystem with the inverse nonlinear dynamic (ID controller) yields the linear dynamics in the integrator-decoupled form

$$y^{(\gamma)} = v \tag{1}$$

which corresponds with a chain of γ integrators between the internal control signal v and the output signal y. All of inherent dynamics of the loudspeaker are compensated and a second subsystem (LD controller) is required to replace the desired linear dynamics into the total system by pole placement. Applied to a voltage-driven woofer system the ID-controller transforms the loudspeaker into a third-order integrator. The preceding LD-controller behaves as an highpass attenuating the low frequency components below the woofer`s resonance frequency (with a slope of 18 dB per octave) and increasing the amplitude of the high-frequency components by almost 6 dB per octave (as long as $L\omega << R_e$). If a loudspeaker is operated over the full audio band the internal control signal v requires substantially increased resolution (additional 10 bit) for the numeric representation of the audio signal. Thus, a loudspeaker controller in the integrator-decoupled form seems not optimal for implementation on low-cost digital signal processors working with a fixed-point format.

The transformation into the integrator-decoupled form is the straightforward way for the derivation of a nonlinear control law but it is shown in this paper that this form is not necessary for exact input-output linearization of affine systems. The control law is presented in a new direct form which dispenses with an additional ID-controller. In a first part of the paper the derivation is illustrated on a vented-box system to expand the application of feedback control to loudspeaker systems with structural radiation aids. The second part of the paper shows the relationship of the derived feedback controller with other control schemes in a state-of-art survey.

2. FEEDBACK CONTROL OF LOUDSPEAKER SYSTEMS

2.1. Electroacoustic Modeling

The derivation of the control law will be illustrated on a vented-box loudspeaker system represented by the analogous circuit depicted in Fig. 2. The driver is represented by the constant lumped elements

- R_e — dc resistance of driver voice-coil,
- m — mechanical mass of driver diaphragm assembly including voice-coil and air load,
- R_m — mechanical resistance of total-driver losses

and the nonlinear elements

- $b(x_p)$ — instantaneous force factor,
- $L(x_p)$ — instantaneous inductance of driver voice-coil, and
- $k(x_p)$ — instantaneous stiffness of the driver suspension

which depend on the voice-coil displacement x_p.

The acoustic system is modeled by the lumped elements

- M_P — acoustic mass of air in the port,
- C_B — acoustic compliance of air in enclosure,
- R_P — acoustic resistance of port losses,

transformed into the mechanical domain in Fig. 2 by the effective projected surface area S_D of the driver diaphragm.

The nonlinear differential equation of the analogous circuit written in the general state space form is

$$\dot{\mathbf{x}} = \mathbf{a}(\mathbf{x}) + \mathbf{b}(\mathbf{x})u$$
$$y = h(\mathbf{x}) \tag{2}$$

where

u is the voltage at the terminals of the voltage driven loudspeaker,

y is the output corresponding to the voice-coil displacement x_p,

$\mathbf{x}(t) = [x_1, x_2, x_3, x_4, x_5]^T = [x_p, dx_p/dt, i, q_P, p_A]^T$ is the state vector of the system comprising displacement x_p and velocity dx_p/dt of the voice-coil, the electrical input current *i,* the volume velocity q_P in the port and the sound pressure p_A in the acoustic system, and

$h(\mathbf{x})$ and the components of $\mathbf{a}(\mathbf{x})$ and $\mathbf{b}(\mathbf{x})$ are smooth nonlinear functions of the state vector $\mathbf{x}$ defined for the vented-box system by

$$\mathbf{a}(\mathbf{x}) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -\dfrac{k(x_1)}{m} & -\dfrac{R_m}{m} & \dfrac{b(x_1)}{m} + \dfrac{L_x(x_1)x_3}{2m} & 0 & -\dfrac{S_D}{m} \\ 0 & -\dfrac{b(x_1) + L_x(x_1)x_3}{L(x_1)} & -\dfrac{R_e}{L(x_1)} & 0 & 0 \\ 0 & 0 & 0 & -\dfrac{R_P}{M_P} & \dfrac{1}{M_P} \\ 0 & \dfrac{S_D}{C_B} & 0 & -\dfrac{1}{C_B} & 0 \end{bmatrix} \tag{3}$$

$$\mathbf{b}(\mathbf{x}) = \begin{bmatrix} 0 & 0 & \dfrac{1}{L(x_1)} & 0 & 0 \end{bmatrix}^T \tag{4}$$

$$h(\mathbf{x}) = x_1 \tag{5}$$

where $L_x(x_1)$ is the first-order derivative of $L(x_1)$.

2.2. Loudspeaker System in Normal State Space Form

According to basic linearizing theory [5], a direct relationship between the input *u* and the output *y* can be established by differentiating the output *y* in respect to time. On the first step one obtains

$$\dot{y} = L_a h + L_b h u \tag{6}$$

where $L_a h$ and $L_b h$ stand for the Lie derivatives of $h(\mathbf{x})$ along the smooth vector fields $\mathbf{a}(\mathbf{x})$ and $\mathbf{b}(\mathbf{x})$, respectively. If $L_b h(\mathbf{x}) \equiv 0$, one continues to differentiate obtaining

$$y^{(i)} = L_a^i h + L_b L_a^{i-1} h u, \quad i = 1, \quad 2, \quad \ldots \tag{7}$$

The differentiation process is terminated if $L_b L_a^i h(\mathbf{x}) \equiv 0$, *for* $i=0, \ldots, \gamma-2$, and $L_b L_a^{i-1} h(\mathbf{x}) \neq 0$ for all $\mathbf{x}$. If the system has a strong relative degree ($\gamma>0$) the input/output dynamics can be concentrated in the equation

$$y^{(\gamma)} = L_a^\gamma h(\mathbf{x}) + L_b L_a^{\gamma-1} h(\mathbf{x}) u = f(\mathbf{x}) + g(\mathbf{x}) u. \tag{8}$$

For the vented-box system the nonlinear functions in Eq. (8) are

$$
\begin{aligned}
f(\mathbf{x}) = &-\frac{k_x(x_1)x_1}{m} - \left(\frac{S_D^2}{C_B} + k(x_1)\right)\frac{x_2}{m} + \left[\frac{b_x(x_1)x_2}{m} + \frac{L_{xx}(x_1)x_2x_3}{2m}\right]x_3 \\
&-\frac{R_m}{m}\left[-\frac{k(x_1)x_1}{m} - \frac{R_m x_2}{m} + \left(\frac{b(x_1)}{m} + \frac{L_x(x_1)x_3}{2m}\right)x_3 - \frac{S_D x_5}{m}\right] \\
&-\left[\frac{b(x_1) + L_x(x_1)x_3}{mL(x_1)}\right]\left[\left(b(x_1) + L_x(x_1)x_3\right)x_2 + R_e x_3\right] + \frac{S_D x_4}{mC_B}
\end{aligned} \tag{9}
$$

and

$$
g(\mathbf{x}) = \frac{b(x_1) + L_x(x_1)x_3}{mL(x_1)} \tag{10}
$$

where $L_{xx}(x_1)$ is the second-order derivative of $L(x_1)$ and $b_x(x_1)$ is the first-order derivative of $b(x_1)$.

Because the relative degree ($\gamma=3$) is smaller than the order ($n=5$) of the vented-box system the input/output dynamics described by Eq. (8) do not give a complete picture of the dynamics in the system. This becomes more obvious by converting the general state space form Eq. (2) into the normal form

$$
\begin{aligned}
\dot{z}_1 &= z_2 \\
\dot{z}_2 &= z_3 \\
\dot{z}_3 &= f\left(\mathbf{T}^{-1}(\mathbf{z})\right) + g\left(\mathbf{T}^{-1}(\mathbf{z})\right)u = f(\mathbf{x}) + g(\mathbf{x})u \\
&\text{-------------------------} \\
\dot{z}_4 &= I_1(\mathbf{z}) \\
\dot{z}_5 &= I_2(\mathbf{z}) \\
&\text{-------------------------} \\
y &= z_1
\end{aligned} \tag{11}
$$

by using the diffeomorphism $\boldsymbol{T}: \boldsymbol{x} \rightarrow \boldsymbol{z}$

$$
\begin{aligned}
z_1 &= T_1(\mathbf{x}) = h(\mathbf{x}) = x_1 \\
z_2 &= T_2(\mathbf{x}) = L_a h(\mathbf{x}) = x_2 \\
z_3 &= T_3(\mathbf{x}) = L_a^2 h(\mathbf{x}) = -\frac{k(x_1)x_1}{m} - \frac{R_m x_2}{m} + \left(\frac{b(x_1)}{m} + \frac{L_x(x_1)x_3}{2m}\right)x_3 - \frac{S_D x_5}{m} \\
z_4 &= T_4(\mathbf{x}) = x_4 \\
z_5 &= T_5(\mathbf{x}) = x_5
\end{aligned} \tag{12}
$$

which is a smooth, locally defined coordinate transformation, that has also a smooth inverse $\boldsymbol{T}^{-1}: \boldsymbol{z} \rightarrow \boldsymbol{x}$.

The new state vector $\boldsymbol{z}$ can be separated in two parts; a first vector $\boldsymbol{z}_o=[z_1, z_2, z_3]^T$ comprises the output and its derivatives and a second vector $\boldsymbol{z}_i=[z_4, z_5]^T$ represents the internal states which are not *directly* controllable from the input and observable at the output. Consequently, the two equations

$$
\dot{\mathbf{z}}_i = \mathbf{I}(\mathbf{z}_o, \mathbf{z}_i) = \begin{bmatrix} \dot{z}_4 \\ \dot{z}_5 \end{bmatrix} = \begin{bmatrix} -\frac{R_B}{M_P}z_4 + \frac{z_5}{M_p} \\ \frac{S_D z_2}{C_B} - \frac{z_4}{C_B} \end{bmatrix} \tag{13}
$$

describe the internal dynamics of the system. The internal stability of the system can be checked by analyzing the zero dynamics

$$\dot{\mathbf{z}}_i = \mathbf{I}(\mathbf{0}, \mathbf{z}_i) \tag{14}$$

that are, the internal dynamics, when the output is controlled to zero by an appropriate input *u*. Clearly, the zero dynamics of the vented-box loudspeaker system are linear and stable.

2.3. Desired Dynamics of the Overall System

With a view of finding the nonlinear control law the dynamics of the overall system (controller + loudspeaker) are defined in normal state space form as

$$\begin{gathered}
\dot{z}_1 = z_2 \\
\dot{z}_2 = z_3 \\
\dot{z}_3 = f_D\left(\mathbf{T}^{-1}(\mathbf{z})\right) + g_D\left(\mathbf{T}^{-1}(\mathbf{z})\right)w = f_D(\mathbf{x}) + g_D(\mathbf{x})w \\
\hline
\dot{z}_4 = I_1(\mathbf{z}) \\
\dot{z}_5 = I_2(\mathbf{z}) \\
\hline
y = z_1
\end{gathered} \tag{15}$$

where

$$\begin{aligned}
f_D\left(\mathbf{T}^{-1}(\mathbf{z})\right) = &-\frac{R_e k(0)}{mL(0)} z_1 - \left(\frac{k(0)}{m} + \frac{b(0)^2 + R_e R_m}{mL(0)} + \frac{S_D^2}{C_B m}\right) z_2 \\
&-\left(\frac{R_e}{L(0)} + \frac{R_m}{m}\right) z_3 + \frac{S_D}{mC_B} z_4 - \frac{R_e S_D}{L(0)m} z_5
\end{aligned} \tag{16}$$

$$g_D\left(\mathbf{T}^{-1}(\mathbf{z})\right) = \frac{b(0)}{mL(0)} \tag{17}$$

and *w* is the controller input signal. This linear system corresponds with the nonlinear model for the vented-box system linearized around ***x=0***. The linearized overall system has also a strong relative degree ($\gamma=3$) which equals the difference between the orders of the denominator and the numerator of the transfer function

$$H_x(s) = \frac{X_p(s)}{W(s)} = \frac{b(0)}{\left(L(0)s + R_e\right)\left(ms^2 + \left(R_m + S_D^2 Z_A\right)s + k(0)\right) + b(0)^2 s} \tag{18}$$

where $X_p(s)$ and $W(s)$ are the Laplace transformed signals $x_p(t)$ and $w(t)$, respectively,

$$Z_A(s) = \frac{M_P s + R_P}{C_B s\left(M_P s + R_P\right) + 1} \tag{19}$$

is the acoustic impedance of the vent in the enclosure and *s* is the Laplace operator.

2.4. Control Law in the Direct Form

Considering the state space representation of the vented-box system in normal form described by Eq. (11) the control law

$$u = \frac{g_D(T^{-1}(\mathbf{z}))}{g(T^{-1}(\mathbf{z}))}\left(w + \frac{f_D(T^{-1}(\mathbf{z})) - f(T^{-1}(\mathbf{z}))}{g_D(T^{-1}(\mathbf{z}))}\right) \tag{20}$$

yields the linearized dynamics in the overall system described by Eq. (15). Expressing the control law as a function of state vector ***x*** by inserting Eqs. (9), (10) and (16), (17) into Eq. (20) and using the diffeomorphism ***T*** given by Eq. (12) the control law in the direct form becomes

$$u = \alpha(\mathbf{x})\left[w + \beta(\mathbf{x})\right] \tag{21}$$

where the control gain is

$$\alpha(\mathbf{x}) = \frac{L(x_1)b(0)}{L(0)\big(b(x_1) + L_x(x_1)x_3\big)} \tag{22}$$

and the control additive is

$$\begin{aligned}\beta(\mathbf{x}) = {} & \frac{R_e}{b(0)}\big(k(x_1) - k(0)x_1\big)x_1 - \frac{L(0)}{b(0)}\left(b_x(x_1) + \frac{L_{xx}(x_1)}{2}x_3\right)x_2x_3 \\ & + \frac{L(0)}{b(0)}\left(k_x(x_1)x_1 + k(x_1) - k(0) + \frac{\big(b(x_1) + L_x(x_1)x_3\big)^2}{L(x_1)} - \frac{b(0)^2}{L(0)}\right)x_2 \\ & + \frac{R_e}{b(0)}\left(\frac{L(0)\big(b(x_1) + L_x(x_1)x_3\big)}{L(x_1)} - b(x_1) - \frac{L_x(x_1)x_3}{2}\right)x_3 .\end{aligned} \tag{23}$$

The feedback controller in the direct form as depicted in Fig. 3 has some advantages over the integrator-decoupled form shown in Fig. 1:

The direct form directly yields the desired behavior in the overall system and compensates only for inherent nonlinearities in the loudspeaker contrary to the integrator-decoupled form where the total dynamics is removed by the ID-controller and the desired linear dynamics is replaced by a separate LD-controller. The control law in the direct form can also be used to modify the linear response by replacing the parameters *b(0), L(0)* and *k(0)* by other desired values.

The direct form causes minimal changes in the transferred audio signal because the control gain becomes $\alpha(\mathbf{x})=1$ and the control additive vanishes $\beta(\mathbf{x})=0$ if the nonlinear loudspeaker parameters equal the linear parameters.

It is also an interesting feature that the mechanic parameters m, R_m and the acoustic parameters C_B, R_B and M_P do not appear in the control law. Thus the identification of these parameters on the particular loudspeaker can be omitted. The internal states x_4 and x_5 are also not required and the direct measurement or estimation of the volume velocity q_P in the port and the sound pressure p_A in the cabinet is not necessary for the linearization of the loudspeaker system.

Furthermore, the whole internal dynamics caused by the vent in the enclosure have no influence on the control law in the direct form. Hence, the control law described by Eqs. (21)-(23) is not restricted to a vented-box system modeled here but gives also exact input-output linearization for loudspeaker systems with other radiation aids (closed box, vented-enclosures with bandpass characteristic, folded horns) as long as the strong relative degree remains constant ($\gamma=3$) and the elements of the radiation aid are linear and can be summarized into the acoustic impedance $Z_A(s)$.

3. RELATIONSHIP TO OTHER CONTROL SCHEMES

3.1. Integrator-Decoupled Feedback Linearization

The controller schemes in the integrator-decoupled form can be converted into the direct form by substituting the equation of the LD-controller which provides the desired pole placement into the control law of the ID-controller. For example the controller developed by Schurer [7, Eqs. (27) and (30)] for the closed-box system can be simplified by converting into the direct form given by Eqs. (21)-(23) and becomes applicable to loudspeaker systems with other radiation aids.

3.2. Feedback Linearization with State Observer

The major problem associated with the practical implementation of feedback linearization is the need for full state measurement. Although the control law in the direct form dispenses from the measurement of the internal states the remaining γ states have still to be monitored by using additional sensors or estimated by a state observer or a state estimator as proposed for loudspeakers by Schurer [7]. The state estimation is based on the loudspeaker input signal u as shown in Fig. 4 and provides the required state information ***x*** by modeling the loudspeaker according to the nonlinear state space Eq. (2). There is a need for precise knowledge of the linear and nonlinear loudspeaker parameters to guarantee stability within the state estimator which is itself a nonlinear feedback system.

The state estimation from the predistorted control signal *u* seems complicated in view of the fact that the linearized overall system already provides a linear relation between the states x_1, x_2 and the control input *w*. However, synthesizing the states from the control input leads to the open-loop filter techniques because there is no state feedback anymore.

3.3. Mirror Filter

Although, the mirror filter [6] is not based on state feedback the control law of the mirror filter is closely related with the control law used in regular static state feedback. To compare both techniques the mirror filter is derived in the notation of feedback control:

Firstly, the nonlinear state equation of the vented-box system on voltage drive given in Eq. (2) is written in the form

$$\begin{aligned} b(x_1)u = R_e L^{-1}\left\{ms^2 + \left(R_m + S_D^2 Z_A\right)s\right\} * x_1 + R_e k(x_1)x_1 + b(x_1)\frac{d\left(L(x_1)x_3\right)}{dt} \\ + b(x_1)^2 x_2 - \frac{R_e}{2} x_3^2 L_x(x_1) \end{aligned} \tag{24}$$

where $L^{-1}\{\}$ is the inverse Laplace transformation, * is the convolution and the acoustic impedance $Z_A(s)$ is given in Eq. (19).

Secondly, the linear overall system defined by Eq. (15) is also converted into the form

$$b(0)w = R_e L^{-1}\left\{ms^2 + \left(R_m + S_D^2 Z_A\right)s\right\} * x_1 + R_e k(0)x_1 + b(0)\frac{d\left(L(0)x_{3l}\right)}{dt} + b(0)^2 x_2 \tag{25}$$

where

$$x_{3l} = L^{-1}\left\{\frac{ms^2 + \left(R_m + S_D^2 Z_A\right)s + k(0)}{b(0)}\right\} * x_1 \tag{26}$$

is the linear input current of a virtual loudspeaker with the properties of the linear overall system. Contrary to the version of the mirror filter presented in [6], Eq. (25) preserves the effect of the constant inductance *L(0)*.

Finally, subtracting Eq. (25) from Eq. (24) yields the control law of the mirror filter

$$u = \frac{b(0)}{b(x_1)}\left\{\begin{array}{l} w + \dfrac{R_e\left(k(x_1) - k(0)\right)}{b(0)}x_1 + \left(\dfrac{b(x_1)^2}{b(0)} - b(0)\right)x_2 \\ -\dfrac{R_e L_x(x_1)}{2b(0)}x_3^2 - \dfrac{d\left(L(0)x_{3l}\right)}{dt} \end{array}\right\} + \frac{d\left(L(x_1)x_3\right)}{dt}. \tag{27}$$

Contrary to feedback linearization the states are synthesized from the input signal *w* by performing the linear filtering

$$x_1 = L^{-1}\left\{H_x(s)\right\} * w \tag{28}$$

$$x_2 = \frac{dx_1}{dt} \tag{29}$$

using the properties of the linear overall system and nonlinear processing

$$x_3 = \frac{b(0)}{b(x_1)}\left\{x_{3l} + \frac{k(x_1 - k(0)}{b(0)}x_1 - \frac{L_x(x_1)}{2b(0)}x_3^2\right\} \tag{30}$$

where $H_x(s)$ and x_{3l} are given by Eqs. (18) and (26), respectively.

The structure of the control law of the mirror filter in Eq. (27) is similar to the structure of the direct control law given by Eq. (21). There is also a control gain which becomes one and additive terms which vanish if the nonlinear parameters equal the linear parameters. Contrary to the direct control law each nonlinear mechanism (parametric excitation, nonlinear stiffness, nonlinear damping, reluctance force, nonlinear inductance) is concentrated here in a separate term. That allows physical interpretation and reduces computational complexity. However, Eq. (27) is not static in the states and requires additional differentiators. The control law of the mirror filter can also be transformed in a form without explicit differentiation. Substituting the linear current state x_{3l} given in Eq. (26) into Eq. (27) and using identities defined by Eq. (2) the control law of the mirror filter becomes identical with the direct control law of feedback linearization given by Eq. (21). Clearly, both control laws are based on the same physical model and provide exact linearization of the modeled nonlinearities in theory.

The major difference between the two approaches is the way of providing the required state information to the control law. The state synthesis in the mirror filter yields an almost complete feedforward structure which is the counterpart of the feedback system repesenting the loudspeaker. This is illustrated in Fig. 5 where the undesired distortion source is separated from the desired part of the nonlinear loudspeaker. The distortion source is just the inverse of the control law in the direct form. Both systems cancel each other out and the state estimation in the mirror filter becomes identical with the state generation of the desired loudspeaker part. It is interesting to see that the linearization of the output signal (displacement) does not allow the linearization of the input current. Thus we need a nonliner system to synthesize the proper input current.

3.4. Mirror Filter with Feedforward Approximation

A control system which has a consequent feedforward structure guarantees stability for any choice of the control parameters because the amplitude of the controller output *u* keeps bounded for any bounded input signal *w.* This property is important for the development of adaptive systems which behaves robust under parameter uncertainties.

The reluctance force of the loudspeaker causes a minor feedback of the nonlinear current x_3 in the last term of Eq. (30). In the practical implementation of the mirror filter in a digital processor it is useful to approximate the exact state Eq. (30) by

$$x_3 = \frac{b(0)}{b(x_1)}\left\{ x_{3l} + \frac{k(x_1 - k(0)}{b(0)} x_1 - \frac{L_x(x_1)}{2b(0)} x_{3l}^2 \right\} \tag{31}$$

where the nonlinear current state x_3 in the reluctance term is replaced by the linear estimate x_{3l} . Alternatively, this reluctance term can completely neglected as done in the original state equation [6, Eq. (15)]. In most practical applications this approximation does not affect the performance of the controller because the reluctance force is usually small in woofer systems.

3.5. Volterra Filter Dedicated to Woofers

The mirror filter with feedforward approximation is closely related with the Volterra filter for woofers proposed by Kaizer [1]. In the original approach the nonlinear transfer response of the loudspeaker is modeled by the Volterra series expansion and the inversion of the mth-order system function yields the control functions of the mth-order homogeneous subsystems.

Alternatively, the control functions can be derived by replacing the nonlinear parameters in the nonlinear control law Eq. (22) or Eq. (27) and in feedforward state Eq. (31) by the power series expansion

$$\begin{aligned} b(x_1) &= b(0) + b_1 x_1 + b_2 x_1^2 + \cdots \\ k(x_1) &= k(0) + k_1 x_1 + k_2 x_1^2 + \cdots \\ L(x_1) &= L(0) + l_1 x_1 + l_2 x_1^2 + \cdots \end{aligned} \tag{32}$$

and assigning the multitude of nonlinear term to the homogeneous subsystems according to their order. It is useful to concentrate the linear state estimation in a separate block preceding the parallel connections of the homogeneous subsystems as shown in Fig. 6. Unfortunately, the computational complexity grows rapidly with the order of the system, thus, higher-order subsystem can not be implemented in available DSP-systems at low costs [7].

3.6. Generic Nonlinear Controllers

The controller schemes discussed here so far are based on a physical modeling of the electrodynamic loudspeaker but these bounded control schemes are not capable of coping with mechanisms which are not considered in the model (e.g. creep and hysteresis of the suspension [8], [9], and the effects of eddy currents [10], [11], [12]).

Nonlinear control architectures developed by other activities in nonlinear signal processing (robotics, image processing, and signal conditioning) are more generic and do not require an exact physical model. Volterra filters in the general form [13] and with reduced complexity [14] have been applied to loudspeakers. Artificial neural networks have also been proposed for loudspeaker controller [15], [16] to realize a general control law.

However, generic controllers have a lower computational efficiency than controllers dedicated to loudspeakers. The number of free controller parameters is usually increased and the parameters are not directly related to a loudspeaker model. Generic controllers have to be adaptive and robust to make the parameter adjustment to the particular loudspeaker feasible.

4. CONCLUSION

The control law in the direct form makes the prefiltering of the audio signal in state feedback linearization superfluous, reduces the computational complexity and requires less loudspeaker parameter than feedback controllers in the integrator-decoupled form. State feedback control of closed-box systems, vented-box systems and many other loudspeaker systems can be accomplished with the same control law if the control law is expressed in the direct form where the states and the parameter of the internal dynamics do not appear. If the remaining states (displacement, velocity, current) are directly measured at the loudspeaker the state feedback linearization dispenses with identification of the acoustic elements and measurement of the internal states in the coupled acoustic system (enclosure, ports, folded horn). Thus, the direct form exploits an important advantage of feedback linearization over other control schemes where the state estimation requires full knowledge of the dynamics.

The control law in the direct form has been derived in this paper from the state space representations of the nonlinear loudspeaker and of the desired overall system in normal form. However, the control law in the integrator-decoupled form can be easily converted into the direct form by unifying the LD-controller with the ID-controller. Hence, there are different ways to derive the direct form.

The control law in the direct form used for regular static state feedback can be transformed into the control law used in the mirror filter. The identity explains why both approaches allow exact linearization of the loudspeaker model. The major differences are in respect to the generation of the required state information. If the direct measurement of the states with a sensor is not practical, state estimation from the controller input signal can be accomplished with higher computational efficiency and improved robustness than state estimation from the controller output. Feedforward estimation of the states also allows implementation of additional delay time in the controller and thus the linearization of nonlinear systems which have not minimum-phase properties (e.g. nonlinear sound propagation in horns [17], [18], [19]).

Although, the feedforward filter techniques have some advantages over feedback linearization the particular application will finally show which technique is optimal in respect to performance, stability, robustness, ability to cope with parameter uncertainties and digital implementation.

5. FIGURES

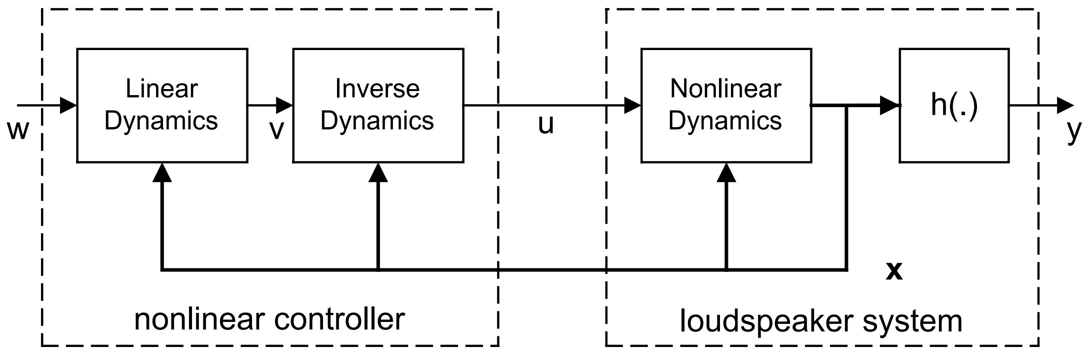

Fig. 1: Regular static state feedback linearization in the integrator-decoupled form.

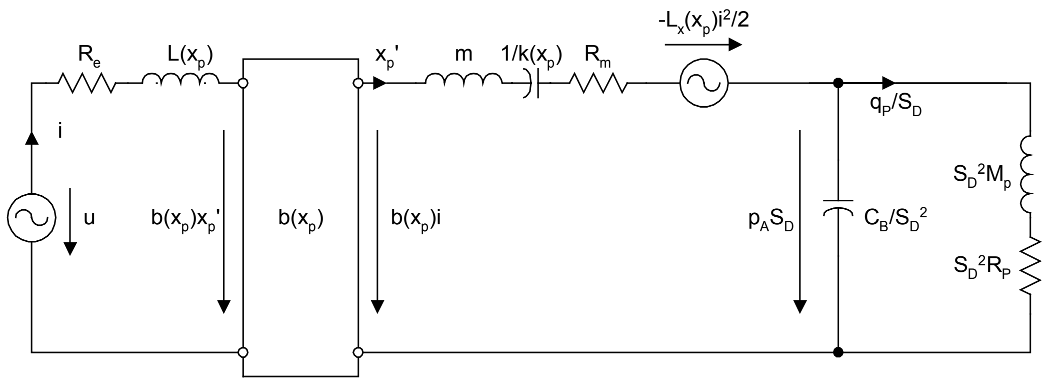

Fig. 2: Mechanical analogous circuit of the vented-box system

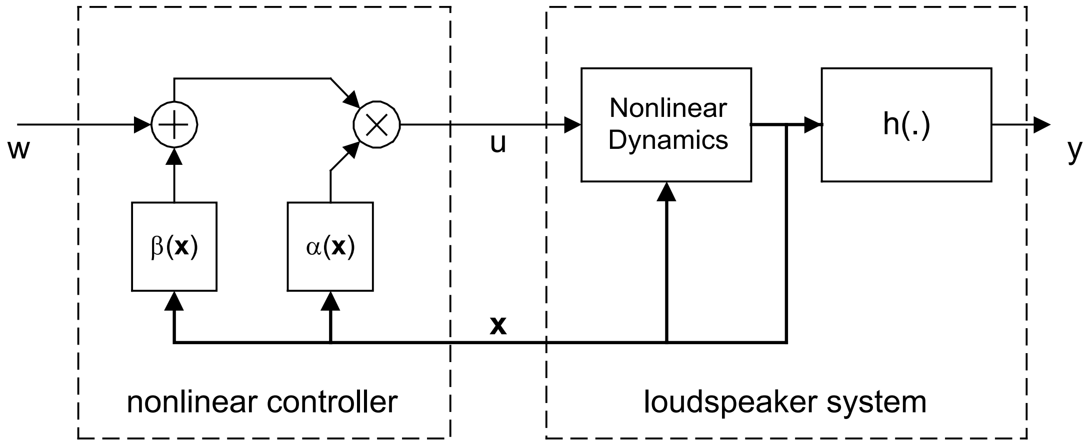

Fig. 3: Regular static state feedback linearization in the direct form

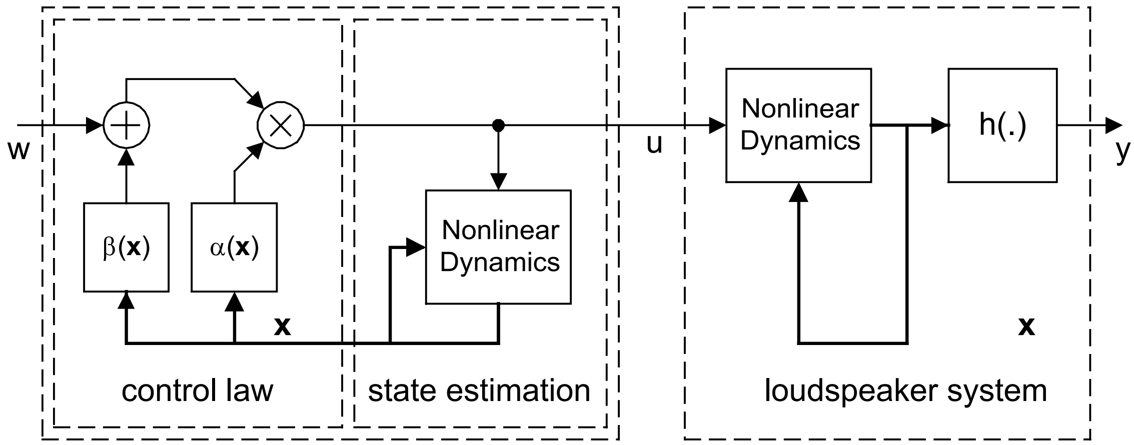

Fig. 4: Feedback linearization with nonlinear state estimation

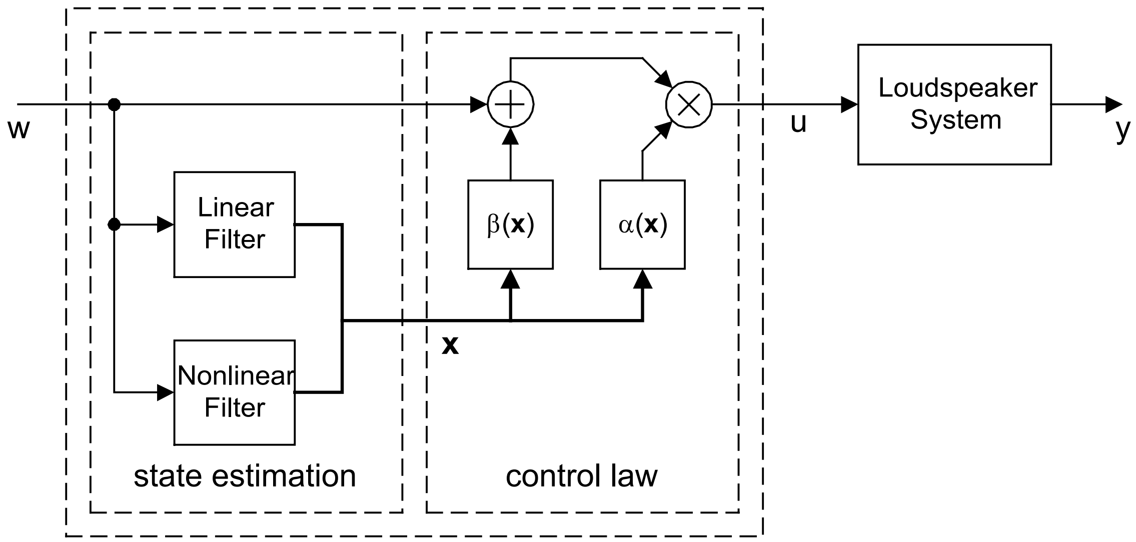

Fig. 5: Mirror filter for woofer systems

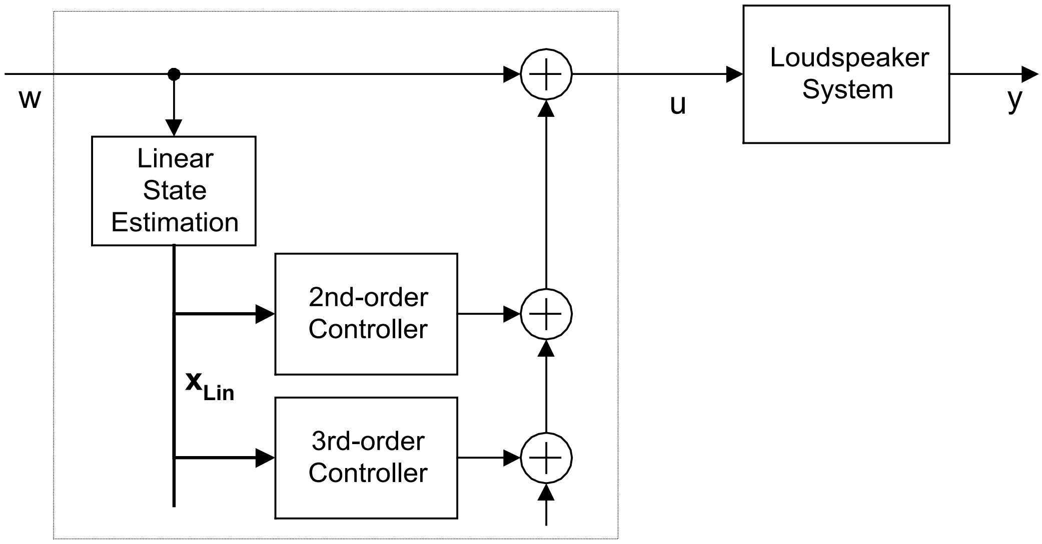

Fig. 6: Volterra filter dedicated to loudspeaker systems

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